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SYSTEMS OF DIFFERENTIAL EQUATIONS OF HEAT AND MASS TRANSFER IN CAPILLARY-POROUS BODIES (REVIEW)

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Abstract—The paper contains a short retrospective review of the works mainly performed by Soviet authors in the field of mathematical description of heat- and mass-transfer phenomena in capillary-porous bodies. The aim of the present work is not only to establish the priority of the scientists working in this field but also to outline the ways of development of the analytical methods for transport phenomena.

Presentation is given of the methods to describe transport phenomena with different transfer potentials, of the relationship between the transfer coefficients and thermodynamic characteristics of capillary-porous bodies.

The limit transitions to the classical Fourier and Fick equations are described. The basic relationships of the extreme cases are illustrated by experimental data. A short description is given of the recent works on turbulent heat and mass transfer in capillary-porous bodies as well as of the methods for transport problems with moving boundaries when the Dirac delta-function and the single Heaviside function are used.

NOMENCLATURE

u,	mass content (moisture content) of a body;
T,	temperature;
P,	total pressure of humid air inside a body;
c,	specific heat of a moist body;
c_0,	specific heat of a perfectly dry body;
c_2,	specific heat of a liquid;
a,	thermal diffusivity of a moist body ($a = \lambda/c\rho_0$);
λ,	thermal conductivity of a body;
ρ_0,	density of a perfectly dry body;
τ,	time;
θ,	mass (moisture) transfer potential;
C_m,	specific isothermal mass capacity of a body;
θ,	temperature coefficient of mass-transfer potential;
r,	specific evaporation heat ($r = r_{12} = r_{21}$);
ε,	phase conversion factor of liquid into vapor;
δ,	thermogradient coefficient based on the moisture content difference ($\delta = \delta_u$);
λ_m,	moisture diffusivity;
M,	molecular mass of humid air;
Π,	porosity;
b,	saturation of body pores and capillaries with moisture;
R,	universal gas constant;
c_p,	isobaric heat capacity;
C_B,	specific body capacity of vapour moisture;
a_p,	convective diffusion coefficient;
k_p,	coefficient of filtrational transfer of vapour moisture;
δ_p,	relative coefficient of filtrational transfer of vapour moisture;
β,	coefficient defined by formula (27).

Subscripts

0,	perfectly dry body;
1,	vapour;
2,	liquid;
3,	air;
4,	ice;
s,	skeleton of perfectly dry body;
(1),	evaporation zone;
(2),	moisture zone;
m,	mass flow ($i = 1, 2, 3, 4$);
K_{ij},	kinetic coefficients ($i, j = 1, 2, 3$);
12,	transition from vapour into liquid (condensation);
21,	liquid evaporation;
w,	surface;
c,	center of a sample.

INTRODUCTION

DUE TO wide use of heat pipes in different branches of technology, transport phenomena in capillary-porous bodies have recently received considerable attention since the wicks of heat pipes are made of such a kind of material [1]. There is a direct inter-

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*Deceased.

relation between liquid and heat transfer in a capillary-porous body since alongside with liquid motion there occurs its enthalpy transfer. This interrelation is also attributed to the fact that liquid is transferred not only under the action of a volumetric liquid concentration gradient but also under that of a temperature.* The interrelation between liquid and heat transfer becomes closer when liquid evaporates inside the porous body. In this case mass inside the capillary-porous body transfers not only in the form of liquid but in the form of vapour as well [3–5]. For this reason, using separately differential equations (Fourier's equation for heat transfer and Fick's equation for mass transfer) the interrelated differential equations are employed for heat and mass transfer in capillary-porous bodies, which were first proposed in [6]. Later on, thermodynamics of moist materials was developed which proved that not liquid concentration (moisture content) but mass-transfer potential θ similar to heat-transfer one (temperature) [7] is shown to be the liquid transfer potential in the capillary-porous body. This fact should be accounted for in the system of two contacting moist bodies. At the border of their contact there occurs a jump of moisture contents (Fig. 1). Moisture content is the analog of heat content (enthalpy) in the theory for heat and mass transfer. As a result, a system of heat- and mass-transfer equations with transfer potentials T and θ was proposed. It was then proved experimentally that with sufficient resistance to vapour motion inside the body there appears the total pressure gradient of humid air which may also account for vapour and liquid transfer. This further complicated the system of differential equations for heat and mass transfer [8]. At present the necessity has arisen to systematize these analytical studies, to outline the ways of simplifying the mathematical presentation of transport phenomena and to show that classical heat-conduction and diffusion equations may be obtained by limit transitions. This necessity is still increased since the differential equations of heat and mass transfer in capillary-porous bodies are used to solve various problems of building thermophysics, drying processes, to study heat and moisture migration in soils and grounds as well as to calculate the wicks of heat pipes, as has been shown earlier.

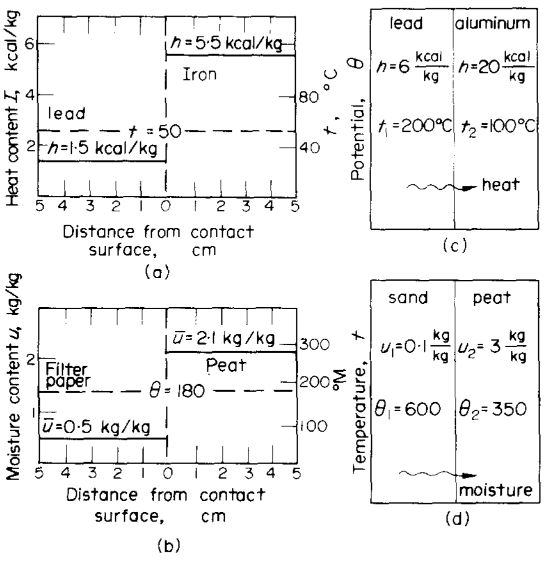

FIG. 1. Distribution of enthalpy (heat content) (a) and moisture content (b) as well as of heat- and mass-transfer potentials (a and b) in the system of two different bodies in contact. Transfer of mass (c) and heat (d) occurs from the larger potential to the smaller one but from the smaller value of heat and mass content to the larger one.

1. TRANSFER EQUATIONS

If u_1, u_2, u_3 and u_4 are used to denote mass capacities of vapour in capillaries, liquid, inert gas and solid material (ice), respectively, and the subscript 0 refers to a dry body, then from the mass conservation law we may write

$$\rho_0 \frac{\partial u_i}{\partial \tau} = -\operatorname{div} j_i + I_i; \qquad i = 1, 2, 3, 4 \tag{1}$$

where j_i is the mass flow density involving diffusional and convective transfer; I_i is the mass sources or sinks of the ith phase due to phase transition, i.e. $\sum_i I_i = 0$. When deriving equation (1) small negligible shrinkage due to mass removal was assumed (body volume remains invariable). If the mass flow in the body is slow, then as an approximation the body skeleton temperature may be considered to be equal to that of any phase of the body material. It follows that vapour in body capillaries is at thermodynamic equilibrium with the liquid.

The energy conservation equation will then be of the form:

$$c\rho_0 \frac{\partial T}{\partial \tau} = -\operatorname{div} q + \sum_i h_i I_i - \sum_i j_i c_i \nabla T; \qquad i = 1, 2, 3, 4 \tag{2}$$

where c is the reduced specific heat of the body

$$c = c_0 + \sum_i c_i u_i \tag{3}$$

c_i is the specific heat of the ith phase mass ($c_i = \mathrm{d}h_i/\mathrm{d}T$); q is the heat flux density.

Consider liquid and vapour transfer in the presence of inert gas (air) in capillaries and pores of the body

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*The two-term formula for non-isothermal mass diffusion in a capillary-porous body was not only proposed by the author in 1935 but also the coefficients of diffusion and thermodiffusion of mass were experimentally determined for a number of moist materials.

($u_4 = 0$), i.e. in the region of positive temperatures.

Since in the inert gas (dry air) no chemical conversions occur ($I_3 = 0$), and in the temperature range above 0°C phase conversions correspond to transition of liquid into vapour, then $I_2 = -I_1$. Here a very important assumption is made: for ordinary conditions of heat and mass transfer and for bodies with maximum porosity, masses of vapour and air cannot exceed 10^{-5} (10^{-3} per cent) of the liquid mass. Mass content of the liquid is, therefore, with high accuracy equal to total mass content, i.e.

$$u = \sum_i u_i \approx u_2; \qquad i = 1, 2, 3. \tag{4}$$

Using relation (4) and making summation in equation (1) with respect to i, we get

$$\rho_0 \frac{\partial u}{\partial \tau} = \sum_i \operatorname{div} j_i = \operatorname{div} j_m. \tag{5}$$

The differential heat- and mass-transfer equations may be obtained from equations (1) and (2) if use is made of the Fourier heat-conduction equation

$$q = -\lambda \nabla T \tag{6}$$

where λ is the total or effective thermal conductivity.

2. At first consider the simple case of no convective or filtrational mass transfer (a zero gradient of the total pressure inside the body). If the mass content and temperature are taken as potentials of diffusional mass transfer, then the expressions for non-isothermal diffusion laws are of the form:

$$j_i = -a_{mi}\rho_0 \nabla u_i + a_{mi}\rho_0 \delta_i \nabla T; \tag{7}$$

$$j_m = \sum_i j_i = -a_m \rho_0 \nabla u + a_m \rho_0 \delta \nabla T. \tag{8}$$

In transient processes the source of vapour mass I_{12} or sink of liquid mass I_{21}

$$I_{12} = -I_{21} = \varepsilon \rho_0 \frac{\partial u}{\partial \tau}. \tag{9}$$

If as an approximation thermodynamic properties (specific heat and thermogradient coefficient) and transfer coefficients (thermal diffusivity and moisture diffusivity) are assumed independent of the coordinates of the body, then when T and u are chosen as transport potentials the differential heat and mass transfer equations will be of the form:

$$\frac{\partial T}{\partial \tau} = a \nabla^2 T + \frac{\varepsilon r}{c} \frac{\partial u}{\partial \tau} \tag{10}$$

$$\frac{\partial u}{\partial \tau} = a_m \nabla^2 u + a_m \delta \nabla^2 T. \tag{11}$$

The thermogradient coefficient δ is based on the moisture content difference ($\delta \equiv \delta_u$).

When drying laminated materials composed of inhomogeneous moist bodies in contact use is made of the system of differential heat and moisture transfer equations with θ, T as transfer potentials, since at the contact of individual layers there occurs moisture content discontinuity (Fig. 1).

The moisture transfer potential θ is the function of the moisture content and temperature of the body

$$\theta = \varphi(u, T) \equiv \theta(u, T). \tag{12}$$

Then,

$$d\theta = \left(\frac{\partial \varphi}{\partial u}\right)_T du + \left(\frac{\partial \varphi}{\partial T}\right)_u dT = \frac{1}{C_m} du + \theta_T dT, \tag{13}$$

where $C_m = (\partial u/\partial \theta)_T$ is the specific isothermal mass capacity (moisture capacity); $\theta'_T = (\partial \theta/\partial T)_u$ is the temperature coefficient of the mass-transfer potential.

Using relation (13) and assuming thermodynamic properties C_m and θ'_T independent of the coordinates and time, the system of equations (10)–(11) takes the form:

$$\frac{\partial T}{\partial \tau} = a' \nabla^2 T + \frac{\varepsilon r C_m}{c'} \frac{\partial \theta}{\partial \tau} \tag{14}$$

$$\frac{\partial \theta}{\partial \tau} = a'_m \nabla^2 \theta + a'_m \delta' \nabla^2 T \tag{15}$$

where the thermodynamic properties and transfer coefficients are equal to:

$$c' = c + \varepsilon r C_m \theta'_T, \qquad \delta' = \frac{\delta}{C_m} + \theta'_T \left(\frac{a'}{a_m} - 1\right) \tag{16}$$

$$a' = \lambda / c' \rho_0, \qquad a' = ac/c', \qquad a'_m = a_m c'/c \tag{17}$$

respectively. Equations (14)–(15) are the same as (10)–(11) differing by the values of heat capacities, diffusion and thermogradient coefficients.

Moisture transfer equation (15) may be obtained from mass conservation equation (5) using the non-isothermal moisture diffusion law

$$j_m = -\lambda \nabla \theta - \lambda_m \delta_\theta \nabla T \tag{18}$$

where δ_θ is the thermogradient coefficient based on the potential difference*

$$\delta_\theta = \frac{\delta}{C_m} - \theta'_T. \tag{19}$$

Relation (19) is obtained from formula (3) when passing from equation (8) to equation (18). Substituting relation (18) into equation (5) and assuming λ_m and δ_θ independent of the coordinates give differential equation (15)

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*The dimensions of the coefficients δ(deg^{-1}) and δ_θ(degM^{-1}) are different, therefore, the coefficient δ'_θ($\delta'_\theta = C_m \delta_\theta$) is introduced in some works. It has the same dimension as δ_θ.

with the same expressions (16)–(17) for the coefficients a'_m and δ'.

In some papers and books the subscripts at the coefficients a', c', a'_m, δ' are omitted, that may result in the wrong idea that the coefficients a, c, a_m and δ in equations (10)–(11) are the same as in equations (14)–(15). It should be borne in mind, which of the transport potentials are used for the description of heat and mass transfer. However, for many moist materials under drying the quantity $C_m\theta'_T$ is small (not the quantity θ'_T itself but rather the product $C_m\theta'_T$), so that $\varepsilon r C_m\theta'_T/c \ll 1$, then we have

$$c' = c, \qquad a' = a, \qquad a'_m = a_m. \tag{20}$$

This is the case when the heat capacities as well as heat and moisture diffusion in equations (10)–(11) and equations (14)–(15) are the same.

If the inequality $\theta'_T(a'_m a_m - 1)/\delta \ll 1$ holds, then $\delta = C_m\delta'$ (the reverse situation corresponds to $\delta_\theta = 0$, i.e. thermal moisture diffusivity may be neglected (moisture transfer is described only by mass-transfer gradient), and we then get

$$C_m\delta' \approx \theta'_T(Lu^{-1} - 1). \tag{21}$$

Once the moisture transfer potential does not depend on temperature ($\theta = 0$), the thermogradient coefficient δ is not equal to zero ($\delta_\theta = \delta/C_m$), since the second term of formula (18) describes thermal moisture diffusion. Similar relations are valid in the case of non-isothermal diffusion in binary gas mixtures when the mass concentration gradient may be expressed in terms of partial pressure and temperature gradients. In addition, thermodiffusion or the Soret effect should be taken into account [7].

3. In case of intense moisture evaporation inside a capillary-porous body the total pressure gradient arises that results in a filtration pattern of vapour transfer. On the basis of the Darcy law the mass flow density j_{fil} in such a situation is defined by the relation

$$j_{\text{fil}} = j_1 + j_3 = -k_p\nabla P. \tag{22}$$

The differential equation governing the pressure field in the body is derived from the balance equation for humid air mass in pores and capillaries of the body

$$\rho_0\frac{\partial(u_1+u_3)}{\partial\tau} = -\operatorname{div} j_{\text{fil}} = \varepsilon\rho_0\frac{\partial u}{\partial\tau} \tag{23}$$

using relation (22).

If as an approximation humid air (vapour–gas mixture) in capillaries and pores of a body is considered to obey the Clapeyron equation of state and swelling of capillary walls to be neglected, then it may be written

$$\rho_0\,\mathrm{d}(u_1+u_3) = \frac{M\Pi b}{RT}\left(\mathrm{d}P - \frac{P}{T}\mathrm{d}T + \frac{P}{b}\mathrm{d}b\right) \tag{24}$$

where b is the saturation of pores and capillaries with vapour and air.

Using relations $\sum_{i=1} b_i = 1$ and $u_2 \approx u$, after simple algebraic manipulations we may obtain

$$\frac{\partial T}{\partial\tau} = K_{11}\nabla^2T + K_{12}\nabla^2u + K_{13}\nabla^2P \tag{25}$$

$$\frac{\partial u}{\partial\tau} = K_{21}\nabla^2T + K_{22}\nabla^2u + K_{23}\nabla^2P \tag{26}$$

$$\frac{\partial P}{\partial\tau} = K_{31}\nabla^2T + K_{32}\nabla^2u + K_{33}\nabla^2P \tag{27}$$

where the coefficients $K_{ij}(i, j = 1, 2, 3)$ are equal to:

$$\begin{gathered} K_{11} = a + \frac{\varepsilon r a_m\delta}{c}, \qquad K_{12} = \frac{\varepsilon r}{c}a_m; \\ K_{13} = \frac{\varepsilon r}{c}a_m\delta_p \end{gathered} \tag{28}$$

$$K_{21} = a_m\delta; \qquad K_{22} = a_m; \qquad K_{23} = a_m\delta_p \tag{29}$$

$$\begin{gathered} K_{31} = a_m\delta\left(\frac{P\varepsilon r}{bT} + \beta - \frac{\varepsilon}{C_B}\right); \\ K_{32} = a_m\left(\frac{\varepsilon rP}{cT} + \beta - \frac{\varepsilon}{C_B}\right) \end{gathered} \tag{30}$$

$$K_{33} = a_p + a_m\delta_p\left(\frac{\varepsilon rP}{cT} + \beta - \frac{\varepsilon}{C_B}\right) \tag{31}$$

where C_B is the specific capacity of vapour moisture (capacity of a capillary body with respect to humid air)

$$C_B = \frac{M\Pi b}{\rho_0 RT} \tag{32}$$

a_p is the convective diffusion coefficient ($a_p = K_p/C_B\rho_0$); β is the coefficient dependent on porosity Π and moisture content u:

$$\beta = \frac{P\rho_0}{\rho_2\Pi - \rho_0 u}. \tag{33}$$

The coefficient accounts for a change in the degree of filling up pores and capillaries of the body with humid air, depending on moisture content of the body; δ_p is the relative coefficient of filtration flow of vapour moisture

$$\delta_p = k_p/a_m\rho_0. \tag{34}$$

When deriving heat-transfer equation (25) an assumption was made that since convection heat transfer in pores and capillaries is small, it may be neglected.

If T, θ and P are taken as moisture transfer potentials, then the system of differential heat- and mass-transfer equations assumes the form:

$$\frac{\partial T}{\partial \tau} = K'_{11}\nabla^2 T + K'_{12}\nabla^2\theta + K'_{13}\nabla^2 P \tag{35}$$

$$\frac{\partial \theta}{\partial \tau} = K'_{21}\nabla^2 T + K'_{22}\nabla^2\theta + K'_{23}\nabla^2 P \tag{36}$$

$$\frac{\partial P}{\partial \tau} = K'_{31}\nabla^2 T + K'_{32}\nabla^2\theta + K'_{33}\nabla^2 P \tag{37}$$

where the coefficients K'_{ij} are equal to:

$$K'_{11} + a' + \frac{\varepsilon r C_m a'_m}{c'}\delta' = a + \frac{\varepsilon r C_m a_m}{c}\delta_\theta; \tag{38}$$

$$K'_{12} = \frac{\varepsilon r C_m a'_m}{c'} = \frac{\varepsilon r C_m a_m}{c};$$

$$K_{13} = \frac{\varepsilon r C_m}{c'} a'_m \delta'_p = \frac{\varepsilon r C_m}{c} a_m \delta'_p; \tag{39}$$

$$K'_{21} = a'_m\delta_\theta + a\theta'_T; \quad K'_{22} = a'_m; \quad K'_{23} = a_m\delta'_p.$$

$$K'_{31} = a'_m\delta_\theta C_m\left(\beta - \frac{\varepsilon}{C_B} + \frac{\varepsilon r P}{cT}\right) + \frac{aP}{T};$$

$$K'_{32} = a_m C_m\left(\beta - \frac{\varepsilon}{C_B} + \frac{\varepsilon r P}{cT}\right); \tag{40}$$

$$K'_{33} = a_p + a_m C_m \delta'_p\left(\beta - \frac{\varepsilon}{C_B} + \frac{\varepsilon r P}{cT}\right).$$

The relative coefficient of filtrational moisture flow δ'_p is based on the moisture diffusivity λ_m

$$\delta'_p = \frac{k_p}{\lambda_m} = \frac{\delta_p}{C_m}. \tag{41}$$

Comparison of the coefficients K_{ij} and K'_{ij} shows that the coefficients K'_{ij} involve an additional factor of specific isothermal mass capacity C_m which is a conversion factor from the moisture content u to the mass-transfer potential. Here it is of importance to note that when deriving the system of equations (35)–(37) the temperature coefficient of moisture transfer potential was not assumed to be equal to zero ($\theta_T \neq 0$). It is then quite natural that thermogradient coefficients $\delta(\delta \equiv \delta_u)$ and δ_θ have different values since these are based on different moisture-transfer potentials at the same temperature drop. Unlike the coefficients δ_p and δ, no direct proportionality between the coefficients δ_θ and δ exists [see equation (19)].

It should be noted once more that sometimes in the system of equations (35)–(37) the prime to the coefficients K_{ij} is omitted but this does not mean that these are equal to the coefficients K_{ij} in equations (25)–(27). It may also be noted that the calculations made by Mikhailov [8] show that the coefficients β and $\varepsilon rP/cT$ are considerably less than ε/C_B for a great number of moist materials, therefore, the expressions for the coefficients K_{3j} may sometimes be simplified; however, experimental determination of these coefficients is the most reliable way.

4. For practical approximate calculations transfer equations (27) and (37) may be simplified. Since mass content of vapour and air in capillaries and pores of a body is negligibly small, as compared to that of liquid ($u_1 + u_3 \ll u_2$), the left hand side of equation (17) may be assumed zero. Then, from equation (23) we have

$$\varepsilon\rho_0\frac{\partial u}{\partial \tau} = k_p\nabla^2 P. \tag{42}$$

The physical significance of this equation is that the total pressure drop inside this body occurs due to liquid evaporation and resistance when vapour moves inside the body (resistance to the filtrational moisture flow).

Relation (42) allows the terms K_{i3} to be eliminated from equations (25)–(27), and system (25)–(27) to be reduced to system (10)–(11), and system (35)–(37), to system (14)–(15). It has therefore made it possible to use the system of equations (10)–(11) or similar system (14)–(15) in drying technology.

In the moist state region more simplifying assumptions may be made. With moisture content of a body greater than the hygroscopic one the vapour pressure of the material is independent of the moisture content but depends only on the temperature (pressure of saturated vapour is a single-valued temperature function). Then with rather intense evaporation of a liquid inside the body the total pressure P inside the body is a function of the temperature only ($P = f(T)$). As an approximation it may be written:

$$\varepsilon\rho_0\frac{\partial u}{\partial \tau} = k_p\left(\frac{\partial P}{\partial T}\right)\nabla^2 T. \tag{43}$$

In this case the differential heat-transfer equation is reduced to the ordinary differential heat-conduction Fourier equation involving the effective thermal conductivity which accounts for heat spent for liquid evaporation inside the body. However, with intense evaporation of a liquid inside the body and large moisture flows the convective contribution to heat transfer inside the body should be taken into account. So, the heat-transfer equation is of the form:

$$\frac{\partial T}{\partial \tau} = \left[a + \frac{r}{c\rho_0}\left(\frac{\partial P}{\partial T}\right)k_p\right]\nabla^2 T - \frac{c_p k_p}{c\rho_0}\left(\frac{\partial P}{\partial T}\right)(\nabla T)^2. \tag{44}$$

5. The phase conversion factor ε is a property of unsteady moisture transfer. It was introduced by relation (9) to characterize a source of vapour moisture. For steady moisture transfer the expression for a source must be different since in this case $\partial u/\partial \tau \to 0, \varepsilon \to \infty$, hence, the source term in relation (9) possesses uncertainty, which may easily be determined. Thus, we

have:

$$I_{12} = -I_{21} = \operatorname{div} j_1 = a_{m1}\rho_0\nabla^2 u + a_{m1}\rho_0\delta_1\nabla^2 T. \quad (45)$$

The quantity $a_{m1}\delta_1$ is the thermodiffusion coefficient of vapour moisture in a capillary-porous body ($a_m^T = a_{m1}\delta_1$). Here the coefficients a_{m1}, δ_1 are assumed independent of the coordinates. Relation (45) is a more general expression for the source of vapour moisture I_{12}, which is also valid for unsteady moisture transfer. Formula (9) is obtained as a specific case from equation (45). As a result, instead of the system of equations (10)–(11), we have the system of differential equations similar to equations (25)–(26) without terms $\nabla^2 P$:

$$\frac{\partial T}{\partial \tau} = K''_{11}\nabla^2 T + K''_{12}\nabla^2 u \quad (46)$$

$$\frac{\partial u}{\partial \tau} = K''_{21}\nabla^2 T + K''_{22}\nabla^2 u \quad (47)$$

where the coefficients $K_{ij}(i, j = 1, 2)$ are equal to:

$$\left.\begin{aligned} K''_{11} &= a + a_{m1}\delta_1\frac{r_{12}}{c} = a + a_{m1}^T\frac{r_{12}}{c};\\ K''_{12} &= a_{m1}\frac{r_{12}}{c}\end{aligned}\right\} \quad (48)$$

$$K''_{12} = a_m\delta, \qquad K''_{22} = a_m. \quad (49)$$

Comparison of equations (48)–(49) with equations (28)–(29) shows that $K''_{21} = K_{21}, K''_{22} = K_{22}$; the coefficients K''_{11} and K''_{12} will be equal to K_{11}, K_{12}, respectively if $\varepsilon = a_{m1}/a_m$ and $\delta = \delta_1$ are assumed (for details of such assumptions, see [7]).

The differential heat- and mass-transfer equations (46)–(47) may thus be used to calculate moisture and heat transfer in capillary-porous bodies with any changes in u and T, including steady processes. The solution of equations (46)–(47) is used in building thermophysics, in chemical engineering as well as in development of experimental methods for measuring thermophysical properties of moist materials. It is quite natural that the system of equations (46)–(47) may be correlated by the additional potential of moisture transfer, P. The system of equations (25)–(27) or (35)–(37) is then obtained, in which instead of the factor ε the coefficients of diffusion a_{m1} and thermodiffusion $a_{m1}^T(a_{m1}^T = a_{m1}\,.\,\delta_1)$ of vapour moisture will be introduced into the formulae for the coefficients K_{ij} and K'_{ij}.

Thus, the differential heat- and mass-transfer equations (25)–(27) or (35)–(37) are the most general ones for diffusional heat and moisture transfer in capillary-porous bodies with the kinetic coefficients which contain coefficients of diffusion and thermodiffusion or filtration transfer of vapour moisture instead of a phase conversion factor. Equation (42) is however used with high accuracy instead of equations (27) and (37).

In this case, when the total pressure gradient is inside the body ($\nabla P \neq 0$), the system of the heat- and mass-transfer equations will contain the coefficients of diffusion a_{m2} and thermodiffusion $a_{m2}\delta_2$ for vapour moisture, i.e.

$$\left.\begin{aligned} \frac{\partial T}{\partial \tau} &= a\nabla^2 T + \frac{\varepsilon r}{c}\frac{\partial u}{\partial r};\\ \frac{\partial u}{\partial \tau} &= a_{m2}\nabla^2 u + a_{m2}\delta_2\nabla^2 T + \varepsilon\frac{\partial u}{\partial \tau}.\end{aligned}\right. \quad (50)$$

Such simplifications are necessary since when the equality ($u \simeq u_2$) is used, the error is negligible. If use is made of the formal mathematical apparatus of irreversible thermodynamics, then such complicated systems of differential equations may be obtained which are of no value for practical calculations [9].

2. LIMITING TRANSITIONS

1. Let during heat and mass transfer the temperature of a moist body be unchanged with time, i.e. $\partial T/\partial \tau = 0$. Then, two cases are possible:

(a) equation (46) implies that $\nabla^2 T = 0$ and $\nabla^2 u = 0$. This is the trivial case of equilibrium state (temperature and moisture content do not vary not only with time but also are independent of the coordinates x_i, i.e. $u = \text{const}$ and $T = \text{const}$) or the case of steady state, for example, in one-dimensional problems the temperature and moisture content are linear time functions; ($\nabla u = \text{const}$; $\nabla T = \text{const}$).

(b) quasi-stationary state at $\partial T/\partial \tau = 0$ from equation (46) it follows

$$\nabla^2 u = -\frac{K_{11}}{K_{12}}\nabla^2 T \neq 0\}^{u^*}_{T^*}\} = f(x); \quad i = 1, 2, 3 \quad (51)$$

(the primes at K_{ij} are omitted, i.e. $K''_{ij} = K_{ij}$, $i, j = 1, 2$), i.e. the fields of both dimensionless moisture content u^* and temperature T^* are similar. Since the form of the function $f(x_i)$ does not depend on time, then according to equation (47) the local velocity $\partial u^*/\partial \tau$ will also be independent of time. In the specific case at $\partial u/\partial \tau = \text{const}$ the distributions of temperature and moisture content in one-dimensional problems are described by a simple parabola. Such a case is encountered in the constant drying rate period when moisture content at any body point is a linear time function ($\partial u/\partial \tau = \text{const}$) and the distribution of moisture content inside the body with symmetric moisture transfer (infinite plate, cylinder, sphere) is described by a parabola. The temperature at any point of the body does not change with time ($\partial T/\partial \tau = 0$). If in the constant drying rate period evaporation occurs only at the body surface (no evaporation inside the body $\varepsilon = 0$ or $a_{m1} = 0$), then the temperature at all points of the body will be the same, and to a first approximation it will be equal to the wet-bulb temperature $T_b (T = T_b = \text{const}, \nabla T = 0, \nabla^2 T = 0)$. This is shown in Fig. 2 when $T_m = 60°\text{C}$; $T_b = 36{\cdot}5°\text{C}$, $\partial u/\partial\tau = 0{\cdot}028$ kg/kg h (2·8%h). This case does not contradict relation (51) since at $\nabla^2 T = 0$; $\varepsilon = 0$ and $K_{12} = 0$, and $K_{11} \neq 0$ and $\nabla^2 u \neq 0$. Indeed, at $\partial T/\partial\tau = 0$ from equation (10) we have that at $\varepsilon = 0$, $\nabla^2 T = 0$. If in the constant drying rate period $(\partial T/\partial\tau = 0, \partial u/\partial\tau = \text{const})$ evaporation occurs inside the body $(\varepsilon \neq 0, a_{m1} \neq 0)$, then according to expression (51) the temperature distribution is described by the parabolic law in such one-dimensional problems since $K_{12} \neq 0$. Such a case is presented in Figs. 3 and 4. All the above cases of moisture content and temperature distributions are observed in the constant drying rate period of colloidal capillary-porous bodies and are at present the experimental facts of experimental laws.

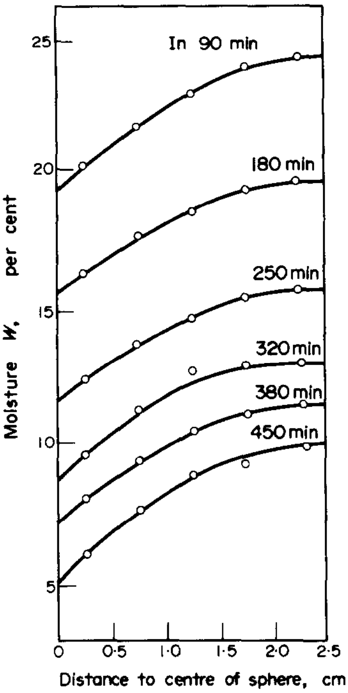

FIG. 2. Moisture content distribution u over the radius of a moist clay sphere in the constant drying rate period at different times ($T_m = 60°\text{C}$; $T_b = 36{\cdot}5°\text{C}$).

2. Let moisture content of a body be invariable with time during heat and mass transfer (quasi-stationary state with heating of a moist body), i.e. $\partial u/\partial\tau = 0$. Then, from equation (47) we have

$$\nabla^2 u = -\frac{K_{21}}{K_{22}} \nabla^2 T, \quad \text{at} \quad \partial u/\partial\tau = 0. \tag{52}$$

If $K_{21} \neq 0$ and $K_{22} \neq 0$, then the field of moisture content of the body is similar to that of the temperature (Fig. 5). However, at $\partial u/\partial\tau = 0$ the local heating rate is not zero $\partial T/\partial\tau \neq 0$ according to equation (46), i.e. the temperature profile inside the body does not vary with time since the temperature behaviour at any body point follows the same law. Such situations may be found in experimental thermophysics, for example, in the method of quasi-steady heating of moist bodies when determining the thermogradient coefficient and thermal conductivity of moist bodies. In this case a moist body is heated with a constant rate (°C/min) when the ambient temperature is a linear time function. Since some time instant the temperature at any body point is a linear time function (Fig. 6) and in one-dimensional symmetric problems the temperature distribution is described by the parabolic law. The

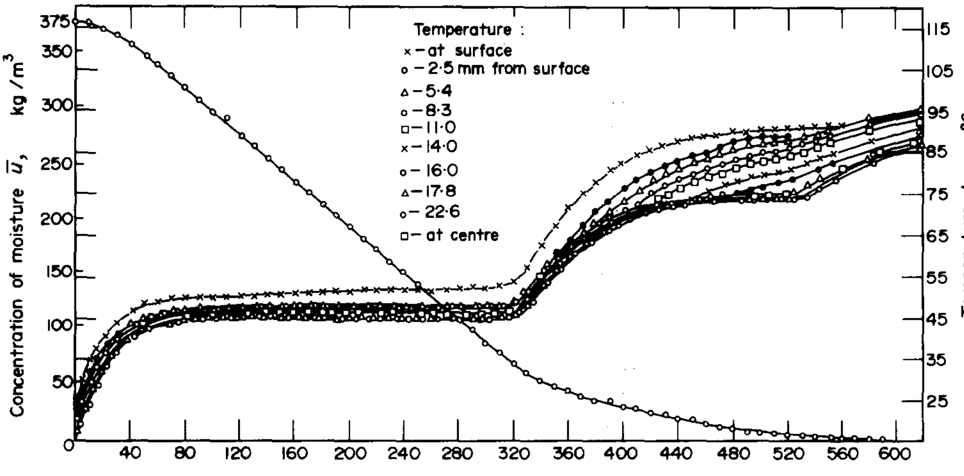

FIG. 3. Drying curve $\bar{u} = f(\tau)$ and temperature curves $T = \varphi(x, \tau)$ of a moist ceramic plate with infrared drying.

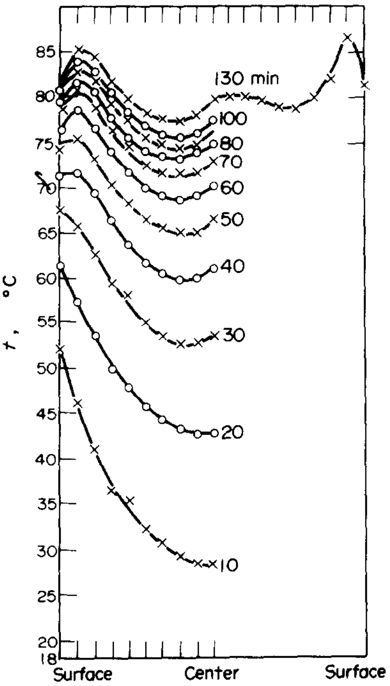

FIG. 4. Temperature distribution across a moist ceramic plate $T = f(x, \tau)$ for the case in Fig. 2.

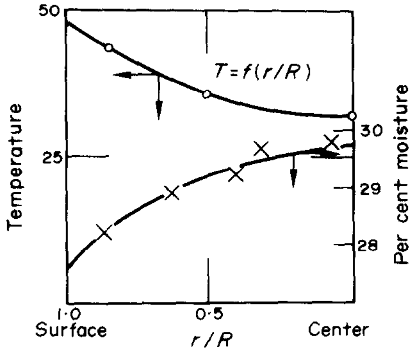

FIG. 5. Change of the temperature with time $T = f(\tau)$ for water (I) and peat (II) in a metallic spherical flask for the co-ordinate $r = 0$ (sphere center).

parabolic law. The parabolic moisture distribution (Fig. 7) corresponds to the parabolic temperature distribution. The coefficients a and δ are determined by the drops in temperature $\Delta T(\Delta T = T_w - T_m)$ and moisture content $\Delta u(\Delta u = u_w - u_m)$, i.e. $T = T_w - (r/R)^2(T_w - T_m)$; $u = u_w - (r/R)^2(u_w - u_m)$ and if heat-transfer coefficient is known, then thermal conductivity may also be found. Figures 6 and 7 show heating of a moist part in a metallic spherical flask 9·94 cm dia in water at a constant rate $b = 0{\cdot}8\,°\text{C/min}$. The temperature drop ΔT is equal to 32°C and the moisture content drop $\Delta u = 14{\cdot}8$. The thermogradient coefficient $\delta = 14{\cdot}8/32 = 0{\cdot}463\%/$

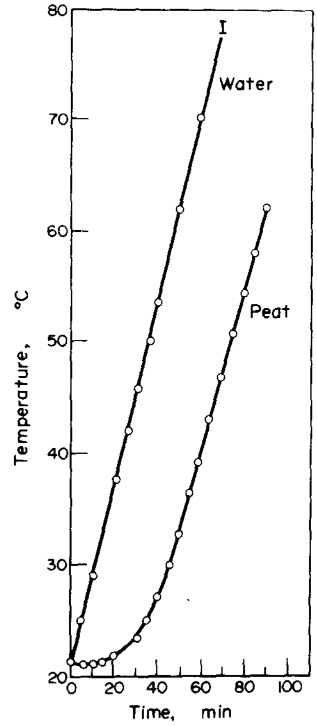

FIG. 6. Distribution of temperature $T = f(r/R)$ and moisture content $u = f(r/R)$ in a clay sphere heated at a constant rate $b = 0{\cdot}37\,°\text{C/min}$ in a quasi-stationary state.

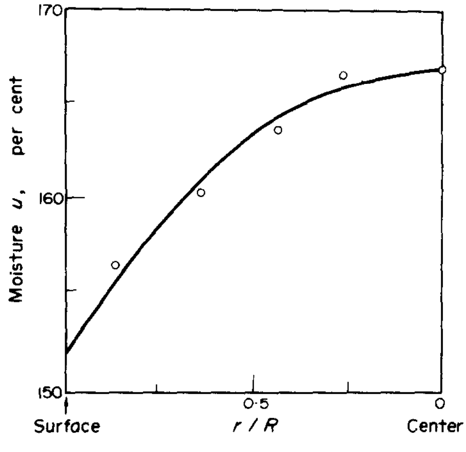

FIG. 7. Moisture content distribution of peat after 80 min heating (Fig. 4) with the uniform initial distribution 158 per cent ($u = u_0 = 1{\cdot}58$ kg/kg).

°C $\approx 0{\cdot}46 \times 10^{-2}$ kg/kg deg. The thermal diffusivity

$$a = \frac{bR^2}{b(T_w - T_m)} = \frac{0{\cdot}8(4{\cdot}98)^2}{6{\cdot}12{\cdot}8}$$
$$= 0{\cdot}26\,\text{cm}^2/\text{min} \approx 4{\cdot}3 \times 10^{-7}\,\text{m}^2/\text{s}.$$

In [2] moisture migration is shown to occur in the form of liquid. Figure 4 shows heating of a clay sphere covered with moisture insulation in humid air at a rate $b = 0{\cdot}37°\mathrm{C/min}$. In this case the coefficient $\delta = 0{\cdot}125 \times 10^{-2}\,\mathrm{kg/kg°C}$. It is quite natural that at $\partial u/\partial\tau = 0$ the trivial case $\partial T/\partial\tau = 0$ may occur at $\nabla^2 u = 0$ and $\nabla^2 T = 0$ (steady state $T = \text{const}$; $\nabla u = \text{const}$ or equilibrium state $u = \text{const}$; $T = \text{const}$). Finally, consider transition of equations (46), (47) into the classical Fourier heat-conduction and Fick diffusion equations. In a perfectly dry body there is no mass transfer (at $u = 0$; $K_{22} = 0$; $K_{21} = K_{12} = 0$). Then, from equations (46) and (47) we get the classical heat-conduction equation

$$\frac{\partial T}{\partial\tau} = K_{11}\nabla^2 T \equiv a\nabla^2 T. \tag{53}$$

The equality $u = 0$ does not imply that according to equation (47) $\nabla^2 T = 0$ since the factor $K_{21} = 0$. The same equation may be obtained from equations (46) and (47) for a moist body with maximum possible moisture content $u_{\max}$ (swelling moisture). Then, at small temperature drops there is no moisture transfer (at $u = u_{\max}$, $K_{22} = K_{21} = K_{12} = 0$), hence, $\partial u/\partial\tau = 0$, and equation (53) is obtained from equations (46) and (47). In this case the moist body is heated in a similar fashion as the dry body with no re-distribution and evaporation of moisture (moisture content at any point is constant and the same $u = u_{\max} = u_{\mathrm{const}}$). If during mass transfer the body temperature is constant, $T = \text{const}$, ($\partial T/\partial\tau = 0$; $\nabla T = 0$; $\nabla^2 T = 0$), then equations (46) and (47) the classical Fick diffusion equation is obtained

$$\frac{\partial u}{\partial\tau} = K_{22}\nabla^2 u. \tag{54}$$

Equation (46) at $\nabla^2 T = 0$ and $\partial T/\partial\tau = 0$ does not imply that $\nabla^2 u = 0$ since $K_{12} = 0$ because $a_{m_1} = 0$ or $\varepsilon = 0$ (no evaporation inside the body).

The experimental data and calculations are specially presented here from the author's works of 1935–1940 to attract the readers' attention to the fact that numerous problems of the theory for transport phenomena in capillary-porous bodies were solved long ago and were obviously unknown to foreign readers, as at present the works appear which deal with these particular problems in the same manner [10–11]. The thermogradient coefficient δ is a counterpart of the thermodiffusion ratio K_T in the theory of thermodiffusion in binary gas mixtures [12–13]. The change in δ with moisture content u for moist capillary-porous bodies (Fig. 8) is similar to that of the thermodiffusion ratio K_T with concentration of the diffusing mixture component. It is important to note that for maximum moisture content ($u \to u_{\max}$) the thermogradient coefficient δ is zero ($\delta \to 0$). Hence, when the body is heated in the case of maximum moisture content, no re-distribution of moisture content due to a temperature gradient occurs. It may also be noted that some relationships of heat and mass transfer in capillary-porous bodies may be used for studying thermodiffusion and diffusion thermoeffect in binary gas mixtures.

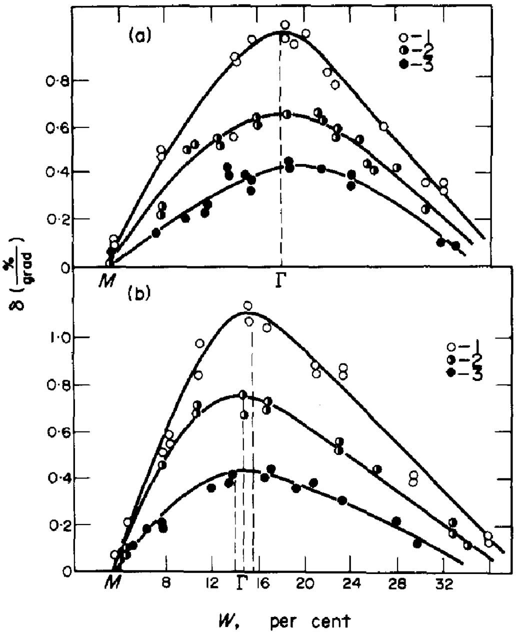

FIG. 8. Thermogradient coefficient δ(%deg) vs clay moisture content u(%); (a) $\rho_0 = 1142\,\mathrm{kg/m^3}$; (b) $\rho_0 = 1000\,\mathrm{kg/m^3}$ at a mean temperature: 1·23°C; 2·33°C; 3·43°C according to [12].

3. INTEGRAL EQUATION

The integral equation of heat and mass transfer may be obtained from the system of equations (46)–(47). If $\bar{T}$ and $\bar{u}$ are used to denote mean volumetric temperatures and mass content, respectively

$$\bar{T} = \frac{1}{TV}\int_{(V)} T\,\mathrm{d}V, \qquad \bar{u} = \frac{1}{V}\int_{(V)} u\,\mathrm{d}V \tag{55}$$

then considering the transport coefficients and thermodynamic properties be invariable over the volume, we have

$$\begin{aligned} V\frac{\mathrm{d}\bar{T}}{\mathrm{d}\tau} &= a\int_{(V)} \nabla^2 T\,\mathrm{d}V + V\varepsilon\frac{V}{c}\frac{\mathrm{d}u}{\mathrm{d}\tau} \\ &= a\int_{(S)} \nabla T\,\mathrm{d}S + V\varepsilon\frac{r}{c}\frac{\mathrm{d}\bar{u}}{\mathrm{d}\tau} \end{aligned} \tag{56}$$

here use was made of the Gauss–Ostrogradsky theorem. Using the boundary condition

$$-\lambda(\nabla T)_w + q_w(\tau) - rj_{2w}(\tau) = 0 \tag{57}$$

where $q(\tau)$ is the heat flux density near the body surface and the differential equation

$$\rho_0\frac{\partial u}{\partial\tau} = -\operatorname{div} j_2(\tau)+\varepsilon\rho_0\frac{\partial u}{\partial\tau} \tag{58}$$

upon slight transformation we have

$$\bar{q}_w(\tau) = r\rho_0 R_V\frac{\mathrm{d}u}{\mathrm{d}\tau}+\rho_0 R_V c\frac{\mathrm{d}\overline{T}}{\mathrm{d}\tau} \tag{59}$$

where $\bar{q}_w(\tau)$ is the heat flux averaged over the surface area, R_V is the ratio of the body volume V to its surface area S ($R_V = V/S$).

Integral equation (59) shows that the heat $\bar{q}_w(\tau)$ supplied to the body is spent for liquid evaporation $r\rho_0 R_V\,.\,\mathrm{d}\bar{u}/\mathrm{d}\tau$ and heating of the body $\rho_0 R_V c\,\mathrm{d}\overline{T}/\mathrm{d}\tau$.

Up to now rather slow motion of liquid and vapour in a capillary-porous body has been considered. The situations are however possible when liquid flows with a high velocity. So, in such a case temperatures of the solid skeleton and of liquid in pores are not the same.

4. INTENSE HEAT TRANSFER

Note that the subscript "s" refers to the solid skeleton and "f", to the liquid flowing inside.

The mean temperature in an infinitesimal volume of a body will then be equal to:

$$\overline{T} = \frac{1}{1-\Pi}\overline{T}_s = \frac{1}{\Pi}\overline{T}_f \tag{60}$$

where Π is the porosity.

Upon using Fourier–Kirchhof's equation separately for the liquid and skeleton and summing them up, we arrive at [15]:

$$[\Pi\rho_f c_f+(1-\Pi)\rho_s c_s]\frac{\partial T}{\partial\tau}+c_f\rho_f\operatorname{div}[\Pi\overline{T}\bar{v}_f]$$
$$= \lambda_f\operatorname{div}(\operatorname{grad}\Pi\overline{T})+\lambda_s\operatorname{div}[\operatorname{grad}(1-\Pi)\overline{T}]+\operatorname{div}q^* \tag{61}$$

where $\bar{v}_f$ is the mean linear velocity of the liquid ($\bar{v}_f = j_f/\rho_f$), q^* is the heat flux vector, in addition to the heat-conduction one

$$q^* = c_f\rho_f[\bar{v}_f\overline{T}_f-\overline{v_f T_f}]+(\lambda_f-\lambda)\frac{1}{V}\int_{S_i}\overline{T}\,\mathrm{d}\mathbf{S} \tag{62}$$

where integration is performed along the surface S_i being in contact with body pores (S is the surface confining the volume V). From equation (61) it follows that unlike Fourier–Kirchhof's equation, heat is transferred not only by conduction

$$q = -[\Pi\lambda_f+(1-\Pi)\lambda_s]\operatorname{grad}\overline{T}, \tag{63}$$

but also by contact transfer between the solid skeleton and liquid (integral term in expression (62)) as well as by turbulent heat transfer (expression in square brackets in (62)). Indeed, the heat flux

$$c_f\rho_f[\bar{v}_f\overline{T}_f-\overline{v_f T_f}] \tag{64}$$

is a counterpart of the turbulent heat flux q_{tur} in moving liquid

$$q_{\text{tur}} = c_p\rho(\overline{T}\bar{\mathbf{v}}-\overline{Tv}) = c_p\rho\overline{T'\mathbf{v}'}. \tag{65}$$

Thus, in a porous body additional heat transfer due to a complex mechanism of liquid motion may exist.

Dwell upon this point in more detail. In most cases turbulent heat transfer is considered proportional to temperature gradient

$$q_{\text{tur}} = c_p\rho_0(\overline{T}\bar{v}-\overline{Tv}) = \lambda_{\text{tur}}\nabla\overline{T} \tag{66}$$

where λ_{tur} is the turbulent thermal conductivity ($\lambda_{\text{tur}} = c_p\rho a_{\text{tur}}$); for liquid flow ($c_p = c_2$; $\rho_2 = \rho$; $\lambda_2 = \lambda$). Use of Fourier–Kirchhof's heat-transfer equation for turbulent incompressible liquid flow ($\rho = \text{const}$) gives

$$c_p\rho\left(\frac{\partial T}{\partial\tau}+\operatorname{div} vT\right) = \operatorname{div}(\lambda\nabla T). \tag{67}$$

If averaging is made with respect to time

$$\overline{T}(\tau) = \frac{1}{\Delta\tau}\int_{\tau}^{\tau+\Delta\tau}T(\tau)\,\mathrm{d}\tau;\quad \bar{v}(\tau) = \frac{1}{\Delta\tau}\int_{\tau}^{\tau+\Delta\tau}v(\tau)\,\mathrm{d}\tau \tag{68}$$

then $\overline{T}$, $\bar{v}$ are different in the time interval $\Delta\tau$. Thus, we have

$$c_p\rho\frac{\partial\overline{T}}{\partial\tau}+c_p\rho\operatorname{div}(\overline{vT}) = \operatorname{div}(\lambda\nabla\overline{T}). \tag{69}$$

Usually

$$\overline{vT} = \bar{v}\overline{T}-\bar{v}\overline{T}+\overline{vT} = \bar{v}\overline{T}-(\bar{v}\overline{T}-\overline{vT}). \tag{70}$$

that results in Fourier–Kirchhof's equation with the additional term $\operatorname{div}(\bar{v}\overline{T}-\overline{vT})$, which is referred to as a divergence of the turbulent heat flux $\operatorname{div} q_{\text{tur}}$

$$c_p\rho\left(\frac{\partial\overline{T}}{\partial\tau}+\bar{v}\nabla\overline{T}\right) = \operatorname{div}(\lambda_{\text{tur}}\nabla\overline{T}). \tag{71}$$

The alternative approach is however possible. $\overline{vT}$ is a vector quantity which may be compared with the vector quantity $\bar{v}\overline{T}$ by means of the second-order tensor **B**

$$\overline{vT} = \mathbf{B}\,.\,\bar{v}\overline{T}. \tag{72}$$

Then, instead of equation (71) we have

$$c_p\rho\frac{\partial\overline{T}}{\partial\tau}+(c_p\rho\nabla\overline{T})\,.\,(\bar{v}\,.\,\mathbf{B})+c_p\rho\operatorname{tr}(\overline{T}\nabla v\,.\,\mathbf{B})$$
$$+c_p\rho\,.\,\overline{T}\,.\,\bar{v}\,.\operatorname{div}\mathbf{B} = \operatorname{div}(\lambda\nabla\overline{T}). \tag{73}$$

Since the liquid is incompressible, $\operatorname{div}\bar{v} = 0$. Equation (73) essentially differs from equation (71) as it contains the turbulent transfer coefficient **B** instead of the turbulent thermal conductivity. From our point of view transition from the vector $\overline{vT}$ to the vector $\bar{v}\overline{T}$ is more correct than use of relations (67) and (70). Similar transformations are possible for relation (62), using formula (72). In this case, however, tensor transformation (72) which establishes the relationship between $\overline{vT}$ and $\bar{v}\bar{T}$ is single but not necessarily linear. The transformation matrix is found from the equation which may be solved using the boundary conditions at the surface A_i. Thus, for capillary-porous bodies introduction of turbulent thermal conductivity allows additional assumptions to be avoided.

The integral relation in formula (64) defines additional heat flux at the contact line of the liquid and the capillary walls. The same situation takes place in case of filtrational liquid motion inside a porous body.

Apart from hydrostatic pressure and other external forces liquid motion inside the porous body involves an additional force exerted by the liquid on the porous structure. The value of the force f_g calculated per unit volume [15]

$$f_g = +\frac{1}{V}\int_{A_i}[\mathscr{P}-p\delta].n^1\,\mathrm{d}A = R\bar{v} \qquad (74)$$

where R is the drag coefficient, which is a function of a local liquid velocity averaged with respect to the volume. The porous body in this case is assumed to be a non-oriented porous structure. The differential equation of filtrational liquid transfer will then be of the form:

$$\nabla p - \eta\,\mathrm{div}(\nabla\bar{v}) + R\bar{v} = 0. \qquad (75)$$

Equation (75) differs from the ordinary Darcy filtrational equation by the additional term Rv. Poiseuille's equation for laminar liquid motion in capillaries is obtained as a specific case ($R = 0$) from the Darcy equation.

5. RECESSION OF PHASE CONVERSION SURFACE

In some cases of heating of moist porous bodies the evaporation surface (surface of phase conversions) may recede. In such a case the problem on heat and mass transfer is mathematically formulated as that with moving boundaries, which may be referred to as an example of the Stefan problem of freezing water or moist body [17]. Solution of problems with moving boundaries affords great difficulties. However, these may be reduced to ordinary heat-transfer problems by introducing equivalent transport coefficients which account for heat of phase transitions. Let us illustrate this on the simplest Stefan problem on freezing of a moist body. The one-dimensional Stefan problem is formulated as follows:

$$c_1\rho_1\frac{\partial T}{\partial\tau} = \frac{\partial}{\partial x}\left(\lambda_1\frac{\partial T}{\partial x}\right); \quad 0 < x < \xi(\tau) \qquad (76)$$

$$T_1(x,0) = \varphi_1(x) \leqslant T_f; \quad 0 < x < \xi(0) \qquad (77)$$

$$T_1(0,\tau) = f_1(\tau) < T_f; \quad \tau > 0 \qquad (78)$$

$$c_2\rho_2\frac{\partial T}{\partial\tau} = \frac{\partial}{\partial x}\left(\lambda_2\frac{\partial T}{\partial x}\right); \quad \xi(\tau) < x < l,\ \tau > 0 \qquad (79)$$

$$T_2(x,0) = \varphi_2(x) \geqslant T_f, \quad \xi(0) < x < l \qquad (80)$$

$$T_2(l,\tau) = f_2(\tau) > T_f, \quad \tau > 0 \qquad (81)$$

where T_f is the freezing surface temperature (for water $T_f \approx$ °C); the subscript 1 refers to the frozen zone (porous body plus ice); the subscript 2 refers to the moist zone (porous body plus water).

The conjugation conditions are

$$T_1[\xi(\tau),\tau] = T_2[\xi(\tau),\tau] = T_f \qquad (82)$$

$$\lambda_2\frac{\partial T_2}{\partial x} = \lambda_1\frac{\partial T_1}{\partial x} = ru_0\frac{\mathrm{d}\xi}{\mathrm{d}\tau}. \qquad (83)$$

As has been shown in [18], this Stefan problem may be reduced to the non-linear heat-conduction problem by introducing the equivalent volumetric heat capacity $\tilde{c}\rho$ defined by

$$\tilde{c}\rho = c\rho + ru_0\delta(T-T_f) \qquad (84)$$

where

$$c\rho = \begin{cases} c_1\rho_1, & T < T_f \\ c_2\rho_2, & T > T_f. \end{cases} \qquad (85)$$

Here $\delta(T-T_f)$ is the Dirac delta function. Then, equations (76) and (79) may be reduced to the single equation

$$\tilde{c}\rho\frac{\partial T}{\partial\tau} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right); \quad \lambda = \begin{cases} \lambda_1, & T < T_f \\ \lambda_2, & T > T_f. \end{cases} \qquad (86)$$

In [18] equation (86) was solved numerically. Good agreement was achieved between the solution obtained and the exact analytical one. In particular, the Stefan law of recession of a freezing surface $\xi = \beta\sqrt{\tau}$ was obtained.

The problem on determination of mass content and temperature fields with evaporation surface recession involves solution of the system of differential heat- and mass-transfer equations with moving boundaries. For a one-dimensional problem it may be written:

$$\left.\begin{aligned} c_{(i)}\rho_0\frac{\partial T_{(i)}}{\partial\tau} &= \frac{\partial}{\partial x}\left(\lambda_{(i)}\frac{\partial T_{(i)}}{\partial x}\right) + \varepsilon_{(i)}r\rho_0\frac{\partial u_{(i)}}{\partial\tau} \qquad (87)\\ \frac{\partial u_{(i)}}{\partial\tau} &= \frac{\partial}{\partial x}\left(a_{m(i)}\frac{\partial u_{(i)}}{\partial x}\right) + a_{m(i)}\delta_i\frac{\partial T_{(i)}}{\partial x} \qquad (88) \end{aligned}\right\}(i) = 1,2$$

where the subscripts (i) denote: $(i) = 1$ is the evaporation zone; $(i) = 2$ is the moist zone.

The conjugation conditions of these bodies are such that at $x = \xi(\tau)$, we have

$$T_{(1)} = T_{(2)}; \qquad u_{(1)} = u_{(2)} \tag{89}$$

$$a_{m(1)}\left[\frac{\partial u_{(1)}}{\partial x}+\delta_{(i)}\frac{\partial T_{(1)}}{\partial x}\right] = a_{m(2)}\left[\frac{\partial u_{(2)}}{\partial x}+\delta_{(2)}\frac{\partial T_{(2)}}{\partial x}\right] \tag{90}$$

$$\left[\lambda_{(1)}\frac{\partial T_{(1)}}{\partial x}-r(1-\varepsilon_{(1)})_{j(1)}(\tau)\right] = \left[\lambda_{(2)}\frac{\partial T_{(2)}}{\partial x}-r(1-\varepsilon_{(2)})j_{(2)}(\tau)\right]. \tag{91}$$

For this problem the Stefan problem may be obtained as a specific case. In the evaporation zone only vapour ($\varepsilon_{(1)} = 1$) is assumed to move while in the moist zone only liquid ($\varepsilon_{(2)} = 0$). Then, invariable mass content of the moist zone is assumed ($u_2 = u = \text{const}$), hence, a mass flow in this zone is absent ($a_{m(2)} = 0$). In the evaporation zone mass content is equal to zero ($u_{(1)} = 0$). From equations (87) and (88) differential equation (66) is then obtained, and from conjugation conditions (90) and (91) condition (83) follows. In this case mass transfer is equal to:

$$j(\tau) = j_1(r) = j_2(\tau) = r\rho_0 u_0\frac{\mathrm{d}\xi}{\mathrm{d}\tau}. \tag{92}$$

If only vapour transfer is assumed to occur in the evaporation zone with no liquid transfer ($\varepsilon_{(1)} = 1$) and only liquid ($\varepsilon_{(2)} = 0$) is assumed to flow in the moist zone, then the phase conversion factor changes spasmodically. Not the temperature of the evaporation zone surface but certain mass content u_f, at which recession of the evaporation surface begins, serves as the boundary of a jump change in the factor. Unlike the Stefan problem on freezing of a moist body, the temperature T_f of the evaporation surface is variable. The evaporation surface is characterized by some mass content u_f below which mass supply becomes less than vapour removal from the evaporation surface, that causes recession of the latter. Then, instead of the systems of differential equations (87) and (88) with moving boundaries it is possible to write an ordinary system of the differential equations

$$c\rho_0\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+r\rho_0\varepsilon\frac{\partial u}{\partial \tau} \tag{93}$$

$$\frac{\partial u}{\partial \tau} = \frac{\partial}{\partial x}\left(a_{m2}\frac{\partial u}{\partial x}+a_{m2}\delta_2\frac{\partial T}{\partial x}\right)+\varepsilon\frac{\partial u}{\partial \tau} \tag{94}$$

in which the phase conversion factor changes spasmodically, i.e.

$$\varepsilon = \varepsilon(u) = [H(u)-H(u-u_f)] \tag{95}$$

here $H(z)$ is the single Heaviside function

$$H(z) = \begin{cases} 0 & z \leqslant 0 \\ 1 & z > 0. \end{cases} \tag{96}$$

Differential mass-transfer equation (94) describes liquid transfer, and vapour transfer is accounted for by the negative source $\varepsilon\partial u/\partial\tau$. The system of equations (93)–(94) is analogous to equation (50) but the phase conversion factor varying spasmodically from 1 to 0 when $u = u_f$ (Fig. 9). However, for some porous materials the phase conversion factor varies with mass content continuously rather than spasmodically (Fig. 10). In this case the factor is a continuous mass content function $\varepsilon(u)$. For approximate calculations the continuous curve $\varepsilon(u)$ may be replaced by the step one (Fig. 10).

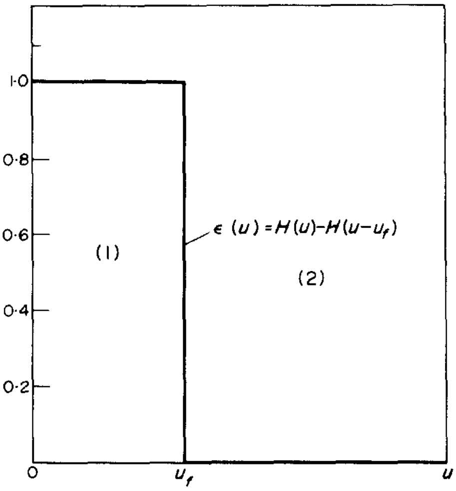

FIG. 9. Step change in ε.

$$\varepsilon(u) \approx H(u)-0{\cdot}7H(u-0{\cdot}04). \tag{97}$$

The advantage of such replacement is that instead of the complex function $\varepsilon(u)$ two limiting values $u_f = 0{\cdot}04$ and $\varepsilon = 0{\cdot}3$ at $u \geqslant 0{\cdot}04$ are chosen.

For a number of porous materials it has been established experimentally that the rate of evaporation surface recession is constant ($\mathrm{d}\xi/\mathrm{d}\tau = \beta = \text{const}$). The complex curve $\varepsilon(x, \tau)$ may then be replaced by the step

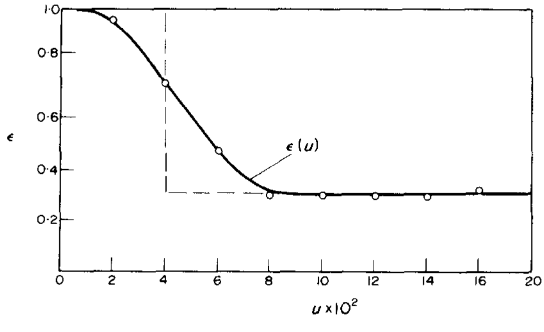

FIG. 10. Plot of factor ε vs moisture content for quartz sand [19].

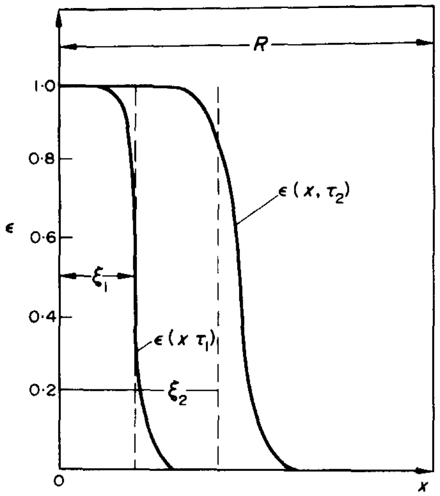

FIG. 11. Relationship between ε and infinite plate coordinate (2R thick) for different times.

one (Fig. 11), and the approximate formula

$$\varepsilon(x, \tau) = H(x) - H(x - \beta\tau) \tag{98}$$

may be used where $H(x)$ is the single Heaviside function. It is convenient to introduce the single Heaviside function when using the integral transformation technique. Thus, the system of differential heat- and mass-transfer equations (93)–(94) may be used to solve the problems on recession of the evaporation surface (problems with moving boundaries). This is due to the fact that when deriving formula (9) for a heat source no limitations are imposed on ε which may be any function of mass content or the function of coordinates and time and may change spasmodically.

Finally, it should be noted that capillary-porous materials are widely used as wicks of heat pipes. Heat pipes allow heat to be transferred in one direction (heat diodes) and thermostating to be performed in the presence of variable heat source. Their operation does not depend on the presence of a gravitational field, therefore, these are widely used in space equipment. For instance, on the space vehicle "Skylab" thermostabilization of the cabin was carried out by means of low temperature heat pipes. At the Heat and Mass Transfer Institute the works are being carried out on application of heat pipes for thermostating of electronic equipment, internal combustion engines as well as for cooling of super-conducting electric cables (cryogenic transmission lines). The current-carrying part of the cable is made of a pack of high-conducting wire nets covered with porous insulation.

For calculation of porous wicks in heat pipes, whose application has been considered earlier, it is necessary to know the basic relationships of heat and mass transfer in capillary-porous materials. Such knowledge was accumulated during the recent decades from investigations of drying processes of moist materials and should be used for developing low temperature heat pipes.

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SYSTEMES D'EQUATIONS AUX DERIVEES PARTIELLES POUR LE TRANSFERT DE CHALEUR ET DE MASSE DANS DES CORPS MICROPOREUX

Résumé—Cet article est une courte revue des travaux principaux d'auteurs soviétiques dans le domaine de la description mathématique des phénomènes de transfert de chaleur et de masse dans des corps microporeux. Le but de ce travail n'est pas seulement d'établir la priorité des travaux dans ce domaine, mais aussi de dégager les voies de développement de méthodes analytiques.

On présente les méthodes pour décrire les phénomènes de transport avec différents potentiels de transfert et on donne la relation entre les coefficients de transfert et les caractéristiques thermodynamiques des corps microporeux.

On précise les limites de transition des équations classiques de Fourier et de Fick. Les solutions pour les cas extrêmes sont illustrées par des résultats expérimentaux. On donne une brève description des travaux récents sur le transfert turbulent de chaleur et de masse dans les corps microporeux et sur les méthodes de résolution des problèmes de transport avec limites déformables.

SYSTEME VON DIFFERENTIALGLEICHUNGEN FÜR WÄRME- UND STOFFÜBERGANG IN KAPILLAR-PORÖSEN KÖRPERN

Zusammenfassung—Die Arbeit enthält einen kurzgefaßten Rückblick über hauptsächlich von russischen Autoren auf dem Gebiet der mathematischen Beschreibung von Wärme- und Stoffübergangsphänomenen in kapillar-porösen Körpern durchgeführten Arbeiten. Ziel der vorliegenden Arbeit ist nicht nur, die Priorität der auf diesem Gebiet tätigen Wissenschaftler zu sichern, sondern auch, die Entwicklungslinien der analytischen Methoden für Transportphänomene aufzuzeigen.

Behandelt werden die Methoden zur Beschreibung von Transportphänomenen mit unterschiedlichen Übertragungspotentialen, die Beziehungen zwischen den Übergangskoeffizienten und thermodynamische Eigenschaften der kapillar-porösen Körper. Die Grenzübergänge auf die klassischen Gleichungen von Fourier und Fick werden beschrieben. Die grundlegenden Beziehungen der Extremfälle werden durch Versuchsergebnisse veranschaulicht. Auf die jüngeren Arbeiten über turbulenten Wärme- und Stoffübergang in kapillar-porösen Körpern wird kurz eingegangen, ebenso auf die Methoden für Transportprobleme mit bewegten Grenzen, soweit die Dirac-Delta Funktion und die einfache Heaviside-Funktion verwendet werden.

СИСТЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ТЕПЛО- И МАССОПЕРЕНОСА В КАПИЛЛЯРНО-ПОРИСТЫХ ТЕЛАХ
(ОБЗОР СОСТОЯНИЯ ВОПРОСА)

Аннотация — В статье представлен краткий исторический обзор, в основном, советских работ в области математического описания явлений тепло- и массопереноса в капиллярно-пористых телах не только с целью установления приоритета ученых, работающих в этой области, но с целью показа путей развития аналитических методов исследования явлений переноса.

Показаны методы описания явлений переноса с разными потенциалами переноса, взаимосвязь между коэффициентами переноса и термодинамическими характеристиками капиллярно-пористых тел. Приведены предельные переходы к классическим уравнениям Фурье и Фикка. Основные закономерности предельных случаев иллюстрированы экспериментальными данными. Кратко описаны последние работы по турбулентному тепло- и массопереносу в капиллярно-пористых телах, а также методы описания задач переноса с подвижными границами на основе использования дельта функции Дирака и единичной функции Хевисайда.